The Formation of Step-Like Structure in Near-Surface and Near-Bottom Pycnoclines

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1. Abstract

A one-dimensional model of unsteady, stratified tidal flow in the presence of background rotation was employed to simulate the formation of step-like density structure under the influence of boundary forcing. Richardson number dependent turbulent diffusivities based on oceanographic microstructure measurements was used for the closure of equations. Upon introduction of a constant surface shear stress, periodic formation and evolution of thin but distinct steps were observed in near surface and near bottom pycnoclines during first three to four cycles of inertial oscillations. The lifetime of these small structures was less than a few hours. Later, several thin quasi-homogeneous layers merge to form larger steps, which became a persistent feature of the mean density profile. A new series of steps was generated thereafter. The details of fine-structure formation were significantly dependent on a combination of factors, including the initial magnitude of the Richardson number, boundary stress, rotation and persistent background shear. It was found that the period between the initialization of boundary forcing and the onset of fine structure is a function of several non-dimensional parameters. The results with tidal forcing exhibited novel features of evolution of step-like structure.

2. Introduction

Well-defined quasi-homogeneous mixed layers separated by narrow sharp density interfaces are common in seasonal oceanic thermoclines and in the pycnocline aloft the bottom boundary layer. *Phillips* (1972) suggested that a stably-stratified turbulent layer can become unstable when the mixing efficiency or flux Richardson number R_f decreases with the gradient Richardson number *Ri* beyond a critical value. Theoretical and numerical studies of this type of instability have been given by *Posmentier* (1977), *Barenblatt et al*. (1993), *Kranenburg* (1996) and *Balmorfth et al*. (1998). *Ruddick et al*. (1989) and Park et al. (1998) showed the possibility of step-like layer formation in series of laboratory experiments. Recently, *Lozovatsky et al.* (1998) emphasized the role of rotation on the generation, formation and evolution of the layered structure. In the present study, we explore the combined effects of wind stress and tidal forcing on the formation of upper and near bottom pycnoclines rich in fine-structure.

3. Model

a) Basic equations

The model (*Lozovatsky and Ksenofontov* 1994) consists of system of equations describing one-dimensional momentum balance (1 and 2), buoyancy transfer (3), and the balance of turbulent kinetic energy (4)

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial z} K \frac{\partial u}{\partial z} + f\left(v - v_T\right) + \frac{\partial u_T}{\partial t}, \qquad \frac{\partial v}{\partial t} = \frac{\partial}{\partial z} K \frac{\partial v}{\partial z} - f\left(u - u_T\right) + \frac{\partial v_T}{\partial t}, \qquad (1), (2)$$

$$\frac{\partial b}{\partial t} = \frac{\partial}{\partial z} K_b \frac{\partial b}{\partial z}, \tag{3}$$

$$\frac{\partial e}{\partial t} = \frac{\partial}{\partial z} K_e \frac{\partial e}{\partial z} + KV_z^2 (1 - Rf) - e , \tag{4}$$

which are closed by a hypothesis (*Lozovatsky et al.* 1993) on turbulent diffusivities

$$K = K_0 / \sqrt{1 + Ri / Ri_{cr}} \quad \text{and} \quad K_b = K / (1 + Ri / Ri_{cr}), \tag{5}$$

where K_b is the mass diffusivity, K is the eddy viscosity and K_0 is defined as

$$K_0 = l_0 \sqrt{e}, \quad l_0 = 1.3 \times \sqrt{e / V_z^2}, \tag{6}$$

where ? is the turbulent kinetic energy per unit mass, t the time, z the vertical coordinate directed downward, $b = g(r - r_0)/r_0$ the buoyancy, r and r_0 are the density and its reference value, g the gravity, f the Coriolis parameter, $K_e = 0.73K$ the turbulent energy diffusivity; $e = 0.1e^2/K$ the kinetic energy dissipation rate, V_z^2 the respective squared mean vertical shear, $Ri = N^2 / V_z^2$ and $R_f = (K_b/K)Ri$ are the gradient and flux Richardson numbers, $Ri_{cr} = 0.1$ and N^2 the squared buoyancy frequency. The Ekman components of the velocity vector u and v are time and depth dependent variables; the components u_T and v_T induced by tidal forces are depth independent.

The closure (5)-(6) implies that the Ozmidov scale

$$L_N = e^{1/2} / N^{3/2} \tag{7}$$

serves as an asymptotic approximation of the turbulence scale l in the case of very strong stratification ($Ri >> Ri_{cr}$). Because the choice of basic turbulent scales can influence the fine-structures predictions of this model, it is of interest to discuss the relationship between the Ozmidov scale and other buoyancy scales that could be used to parameterize vertical mixing in stably stratified flows.

b) Turbulent scales

Oceanic microstructure can be characterized by velocity, length and time scales of motions. In a stably stratified water column, eddies with characteristic rms velocity $e^{1/2}$ and length scale l can overturn against the stable stratification, provided that the inertia forces of eddies e/l exceed the corresponding buoyancy forces $N^2 l$, or when $l << L_b$, where $L_b = e^{1/2}/N$. As pointed out by *Hunt* (1985), L_b is a rational choice for the turbulence lengthscale in stratified non-sheared flow. Kinematically based lengthscales that have been used for stratified turbulence include the Ellison scale $L_E = Q_b / N^2$, where Q_b is the rms of buoyancy fluctuations, and the Thorpe scale L_{Th}. The latter signifies the scale of overturning in a turbulent region and is calculated by reordering a density profile containing inversions to obtain a stable profile with monotonic variation of density. Laboratory experiments of *DeSilva and Fernando* (1992) show how L_{Th} of a growing turbulent patch increases with time and then achieves a maximum value proportional to L_b. Recently *Lozovatsky and Fernando* (2000) argued that the Thorpe scale, in either growing or decaying patches, is a function of the mixing Reynolds number and patch Richardson number. The rapid generation of a turbulent layer, its growth when the turbulence scale $l << L_b$ or L_N (active turbulence), the onset of buoyancy effects distinguished by the retardation of growth at $l = L_b$ and the subsequent decay of l in equilibrium with decaying L_N are the essentials of *Gibson's* (1980) fossil turbulence theory. When the overturning scale $l \sim L_N$ decays to the Kolmogorov scale $h = (n^3/e)$, the dissipation rate decreases to $e \le (25 - 30) \times nN^2$; here n is molecular viscosity, and the flow becomes completely fossilized.

If the length scale that characterizes the shear production $L_{sp} = l_0 / 1.3 = \sqrt{e/V_z^2}$ is larger than L_b, that is if $Ri >> 1$, then the length scale demarcating between the inertial subrange and the large scale of overturning motions becomes L_b, and given that $e \sim e^{3/2} / L_b$, it is possible to write $L_b \sim L_N$. On the other hand, if $Ri << 1$, then $L_{sp} << L_b$, and $e \sim e^{3/2} / L_{sp}$. In this case, the Ozmidov scale cannot be directly used to interpret stratified turbulence. An alternative buoyancy scale, known as the Bolgiano-Obukhov scale L_*,

$$L_* = e^{5/4} c_T^{-3/4} (g a_T)^{-3/2} , \tag{8}$$

where c_T is the temperature (or other buoyancy-related scalar) dissipation rate, a_T the coefficient of thermal expansion has also been used in stratified turbulence studies. The derivation of this lengthscale assumes the existence of a wavenumber subrange in stably stratified spectra where the turbulence is locally axi-symmetric and horizontally homogeneous and large eddies are affected by buoyancy.

To establish a relationship between the Ozmidov L_N and Bolgiano-Obukhov L_* scales, assume that the buoyancy fluctuations are produced by turbulent motions within a stratified layer at the scales on the order of L_{pr}. If so, the buoyancy variance $\overline{Q_b^2} = \overline{(g r' / r_o)^2}$, which relates to the temperature variance $\overline{Q_T^2}$ as

$$\overline{Q_T^2} = \overline{Q_b^2} / (g a_T)^2 , \tag{9}$$

can be defined as

$$\overline{Q_b^2} = N^4 L_{pr}^2 . \tag{10}$$

The temperature dissipation c_T in the buoyancy subrange can be parameterized in terms of the turbulent kinetic energy e, the variance of temperature $\overline{Q_T^2}$ (or buoyancy $\overline{Q_b^2}$) fluctuations, and the production scale L_{pr} as

$$c_T = c_T \frac{\sqrt{e_{tr}}}{L_{pr}} \overline{Q_T^2} , \tag{11}$$

where c_T is a non-dimensional constant.

Substitution of (9), (10) and $e = c_e e^{3/2} / L_{pr}$ in (11) gives

$$c_T = c_c \frac{N^4 e^{1/3} L_{pr}^{4/3}}{(g a_T)^2} , \; c_c = \frac{c_T}{c_e^{1/3}} . \tag{12}$$

If we use (12) as the temperature dissipation in the buoyancy subrange and substitute it in (8), the following relationship can be obtained between the Bolgiano-Obukhov, Ozmidov and production lengthscales

$$L_* = c_L L_N^2 / L_{pr} , \tag{13}$$

where

$$c_L = a_* \left(c_e / c_T^3 \right)^{1/4} . \tag{14}$$

If the scale of turbulent production L_{pr} is proportional to the Ozmidov scale L_N, (14) suggests possible linear dependence between L_* and L_N. A test of this hypothesis is shown in Fig. 1 using the microstructure measurements taken on a shallow Black Sea shelf (*Lozovatsky et al*. 1999). Although there is wide scatter of individual samples, correlation between L_N and L_* is clearly evident. The best least-square fit gives a linear relation between Ozmidov and Bolgiano-Obukhov scales as

$$L_* = 2.92 \times L_N . \tag{15}$$

Peters et al. (1995) found that $c_e \approx 4$ and $c_T \approx 7$ for turbulent patches in the equatorial Pacific and *Lozovatsky* (1987) obtained $a_* = 2.7$. Substitution of these values in (14), gives $c_L \approx 0.88$, which is much smaller than that obtained in (15). This discrepancy can be reconciled by assuming $L_{pr} \approx 0.3 \times L_N$. Since the production scale is usually associated with the Thorpe displacement scale for turbulent overturns, or with a shear production scale $L_{sp} = e^{1/2} / V_z^{3/2}$, L_{sp} is related to L_N as

$$L_N = Ri^{-3/4} \times L_{sp}\,, \tag{16}$$

and since $L_{sp} \approx 0.3 \times L_N$, the local Richardson number can be established as $Ri \approx 0.2$. This is in agreement with the observations made in stratified inner layer on the Black Sea shelf (*Lozovatsky and Fernando*, 2000).

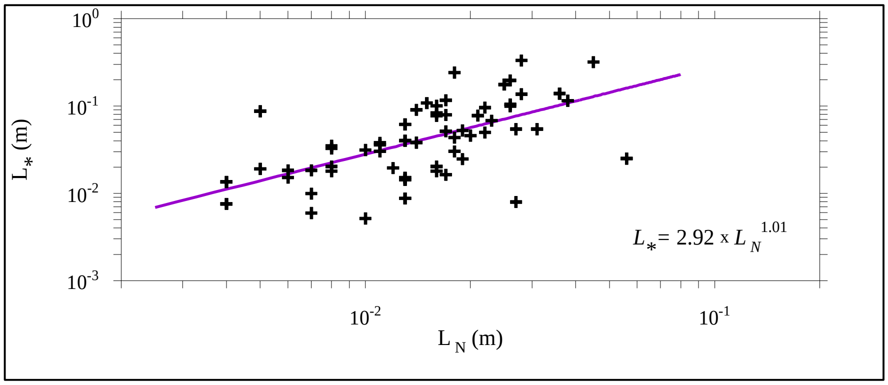

Fig. 1. The dependence between Ozmidov L_N and Bolgiano-Obukhov L_* scales in a stratified layer on the Black Sea shelf.

The discussion given above shows that the use of Ozmidov scale in our numerical model is a reasonable approximation for the turbulent lengthscale l in strongly stratified layers. The scale L_{sp} works mainly in homogeneous layers and the intermediate scale $L_R = e^{1/2} / N V_z^{1/2}$ is a key player in the transition region between quasi-homogeneous boundary layers and neighboring pycnoclines (*Lozovatsky*, 1996).

c) Dependence on the initial and boundary conditions

The following initial and boundary conditions were imposed in solving Eqs. (1) - (6):

$$t = 0: \quad u = u^{o}(z), \quad v = v^{o}(z), \quad r = r^{0}(z), \quad e = e^{0}(z), \quad u_T = u_A{}^{o}\sin(w_T t - j_1), \quad v_T = v_A{}^{o}\sin(w_T t - j_2) \tag{17}$$

$$z = 0: \quad K\frac{\partial u}{\partial z} = \frac{t_x}{r_0} = u_*^2, \quad K\frac{\partial v}{\partial z} = 0, \quad K_b\frac{\partial b}{\partial z} = 0, \quad K_e\frac{\partial e}{\partial z} = -P_e, \tag{18}$$

$$z = H: \quad u = v = 0, \quad u_T = v_T = 0, \quad K_b\frac{\partial b}{\partial z} = 0, \quad K_e\frac{\partial e}{\partial z} = 0 \tag{19}$$

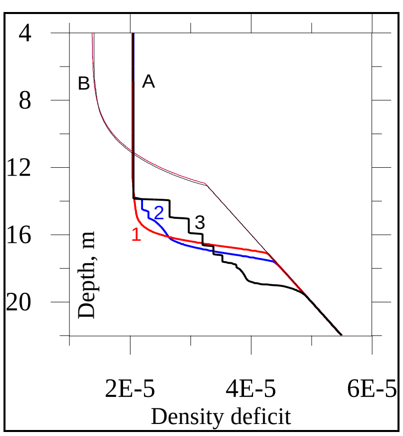

Fig. 2. ?h? influence of V_{zin} (see text).

Here $z = 0$ is the sea surface and $z = H$ is the bottom boundary. In solving the equations, stable conservative numerical schemes with high order approximation for spatial and temporal derivatives satisfying conservation laws were used. The grid step of 2 cm and time step of 4 min were chosen to achieve fine scale resolution for hydrophysical profiles.

The closure hypotheses (5)-(6) require vertical shear to be non-zero at any depth during all time of the calculation. Therefore, a sustained "background" shear V_{zin} was introduced, which imposed specific hydrodynamic conditions on modeling. The development of vertical profiles under the influence of boundary forcing in this model significantly depends on the non-dimensional parameters f/N_{in} and on $Ri_{in} = N_{in}^2 / V_{zin}^2$, which is the initial Richardson number. If $Ri_{in} >> Ri_{in}^{cr}$ for a given f/N_{in}, the pycnocline formed below the upper quasi-homogeneous layer does not contain any fine structure (Fig.2, curve 1, set A, no tidal forcing). For $Ri_{in} \le Ri_{in}^{cr}$, a few prominent steps are generated in the pycnocline after 60 hours of the onset of calculation (curve 2). For small Ri_{in}, sudden generation of step-like structure started after 36 hours of calculations (curve 3). The influence of V_{zin} on the development of the pycnocline started after a few hours of boundary mixing from the initiation of the model, when the wind induced current shear decreased below V_{zin}. Before this, up to t = 10 h, all profiles evolved almost identically (Fig. 2, set B) and only later they depart from each other. A critical time t_{cr} at which the background shear begins to play an important role in turbulence generation appears to be a function of the ratios u_*^2/e^0V_{zin} and f/N_{in}. An explicit form of this dependence has not been established yet. The actual value of the initial Richardson number, which is critical for layer formation, can differ depending on the combination of u_*^2/e^0V_{zin} and f/N_{in}. For calculations shown in Fig. 2, $Ri_{in}^{cr} \approx 6$. The initial and boundary conditions for calculations shown in Fig. 2 were the same as those specified in the next section.

4. Tidal Current Experiment

The initial linear density profile in these calculations was set as $r^0(z) = r_0 + z(dr/dz)_{in}$ with the constant density gradient $(dr/dz)_{in} = 2.5\times10^{-3}$ kg/m^4 and r_0 = 1025 kg/m^3; the corresponding initial $N_{in}^2 = 2.45\times10^{-5}$ s^{-2}. The “background” shear $V_{zin} = 2\times10^{-3}$ s^{-1} ensured that $Ri_{in} = N_{in}^2/V_{zin}^2 = 6.1$. Surface wind stress and turbulent energy flux were calculated using the bulk formulae corresponding to a constant wind speed of 8 m/s; $u_* = 9.2\times10^{-2}$ m/s. The amplitudes of semidiurnal w_T = 2π/12.4 hr^{-1} tidal component are 5 cm/s, the Coriolis parameter $f = 10^{-4}$ s^{-1} and the water depth is 42m. The results of calculations are shown in Fig. 3, 4 for the time period 15 to 180 hours after the onset of winds.

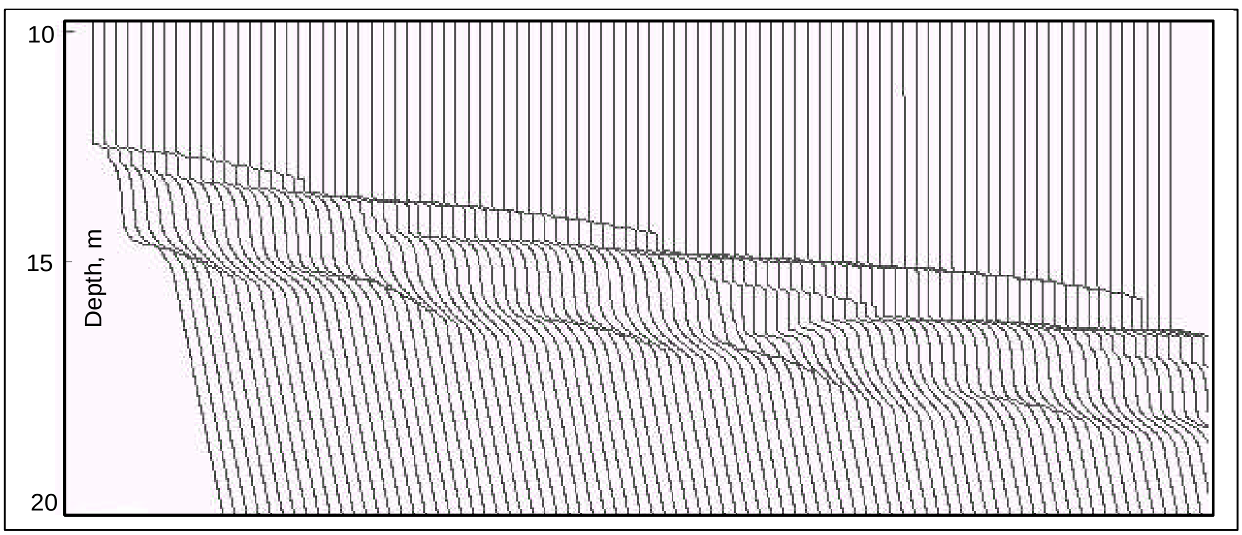

Fig. 3. Formation of the upper pycnocline under constant wind stress.

The model clearly reproduces an upper well-mixed homogeneous layer, which gradually deepens under sustained wind stress. The fine structure in the pycnocline is modulated by the periodic amplification of vertical shear associated with inertial oscillations. Phases of the generation of new steps also change with periods of thin-layer merging. As a result, a pycnocline with persistent fine structure is formed. In the bottom layer (Fig. 4), where tidal friction produces vertical mixing near the very bootom, the development of turbulent boundary layer is limited by inertial - buoyancy balance. The height of the homogeneous near bottom layer is stabilized after about 140 hours of calculations, achieving almost 8-m thickness. The pycnocline aloft this mixed layer is somewhat weak in fine structure, compared to the upper part of the water column affected by the wind stress. The interaction between the tidal t_T = 12.4 h and inertial t_f = 17.4 h oscillations generates long period beats with frequency $w_{bt} = w_T - w_f$.

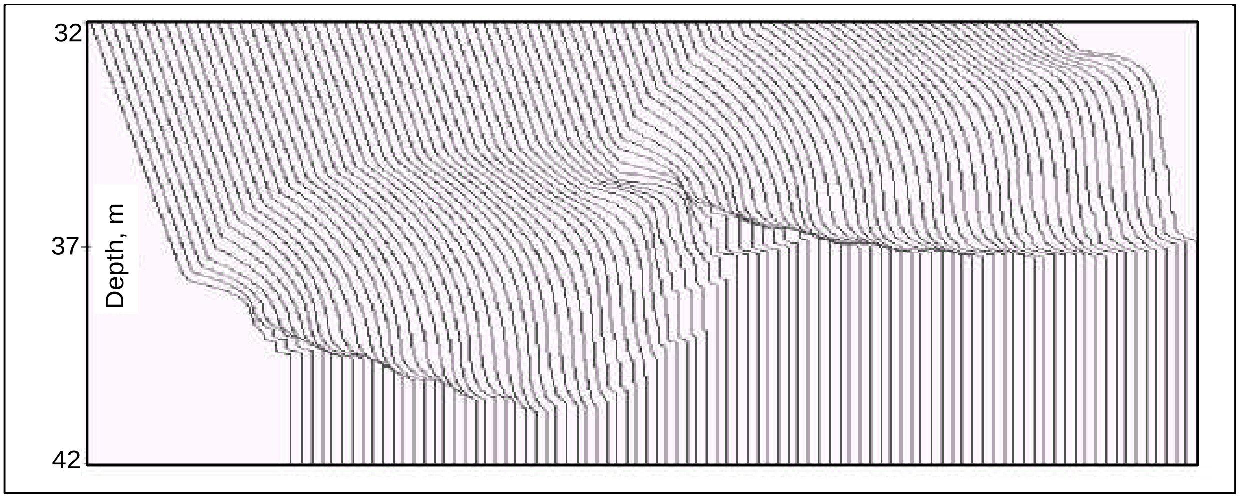

Fig. 4. Evolution of the near bottom mixed layer and pycnocline aloft in the presence of a tidal current.

The evolution of the homogeneous layer and density gradients in the near bottom pycnocline are strongly influenced by such beats that have a period of $t_{bt} \approx 42$ h. The present version of model does not contain any parameterization for the bottom roughness, and accounting for it will be the next step of our efforts.

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